Disciplined Geometric Programming
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Domain-specific languages (DSLs) for convex optimization
- DSLs for convex optimization make it easy to specify and solve convex problems.
- Modern DSLs (CVXPY, CVXR, Convex.jl, CVX) based on disciplined convex programming (DCP) [6]
- DCP is a library of functions (atoms) with known curvature and monotonicity, and a composition rule for combining them.

Hierarchy of optimization problems.
LLCPs generalize GPs and so-called generalized geometric programs (GGPs)

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<th>Properties</th>
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<td>Convexity with respect to the geometric mean, $f$ is log-log convex if (and only if) it is convex with respect to the geometric mean, i.e., $f(x^\theta \circ y^{1-\theta}) \leq f(x)^\theta f(y)^{1-\theta}$, for $\theta \in [0, 1]$ and $x, y \in \mathbf{dom} f$ (o is the elementwise product, and the powers are meant elementwise)</td>
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Examples

Log-log affine functions
- products
- ratios
- powers

Log-log convex functions
- $x_1 + x_2$, $\max(x_1, x_2)$
- posynomials
- $f_p$ norms
- $(|X - X|)^{-1}$, (spectral radius of $X$ less than 1)

Log-log concave functions
- $x_1 - x_2$, with $x_1 > x_2 > 0$
- $-x \log x$, $x \in (0, 1)$
- complementary CDF of a log-concave density, e.g., $\frac{1}{\sqrt{2\pi}} \int e^{-x^2/2} dt$

Disciplined geometric programming
Analog of DCP, but for LLCPs
- Library of atoms with known log-log curvature (sum, product, ratio, exp, ...)
- Atoms may be combined using the composition rule
- Can express LLCPs of the form

$$\begin{align*}
\minimize & \quad f_0(x) \\
\text{subject to} & \quad f_i(x) \leq g_i(x), \quad i = 1, \ldots, m \\
& \quad g_i(x) = \bar{g}_i(x), \quad i = 1, \ldots, p
\end{align*}$$

with $f_i$ log-log convex, $\bar{f}_i$ log-log concave, $g_i$ and $\bar{g}_i$ log-log affine (must be verifiable by the composition rule)

Implementation
- DGP implemented as a reduction in CVXPY 1.0:
  https://www.cvxpy.org/tutorial/dgp/index.html

Examples

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References